

26-02-2019.

Tuesday.

Magnetosonic Wave:-

It means sound wave in the presence of magnetic field.

• Sound waves in the presence of magnetic field is called magnetosonic waves.

• It is electromagnetic wave because "E & k" are \perp to each other.

• "E" is along x-axis, "k" is along y-axis & B_0 is along z-axis.

• Ions plays important role in low frequency waves.

• ∇P_i is along the y-axis.

• Due to drift, compressions and rarefactions are expected so, we take thermal pressure ∇P_i along y-axis.

Equation of motion for ions:-

$$m_i n_{i0} \frac{\partial v_{ix}}{\partial t} = n_{i0} e [E_x + v_{iy} B_0] - \frac{\nabla P_i}{n_{i0}}$$

$$m_i \frac{\partial v_{iy}}{\partial t} = e [E_y + v_{ix} B_0] - \frac{\nabla P_i}{n_{i0}}$$

X-component:-

$$\frac{\partial v_{ix}}{\partial t} = \frac{e}{m_i} (E_x + v_{iy} B_0)$$

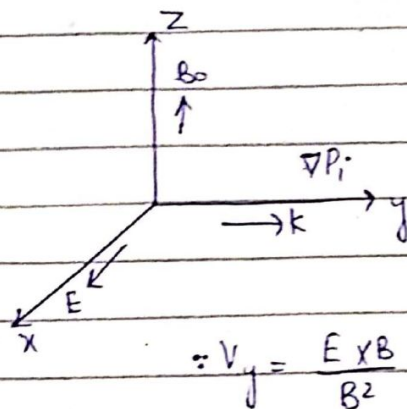
$\therefore \nabla P_i = 0$ because it is along y-axis.

$$v_{ix} = \frac{ie}{m_i \omega} (E_x + v_{iy} B_0) \quad \text{--- (1)}$$

Y-component:

$$\frac{\partial v_{iy}}{\partial t} = \frac{e}{m_i} (0 - v_{ix} B_0) - \frac{1}{m_i n_{i0}} \gamma_i k_B T_i \nabla n_i$$

$$v_{iy} = \frac{-ie B_0 v_{ix}}{m_i \omega} + \frac{\gamma_i k_B T_i}{m_i n_{i0}} \frac{1}{\omega} \nabla n_i$$



$$\therefore v_{th}^2 = \frac{2k_B T}{m}$$

$$V_{iy} = -\frac{i\omega a_i}{\omega} V_{ix} + \frac{\gamma_i k_B T_i}{m_i} \frac{k}{\omega} \frac{n_{i1}}{n_{i0}} \quad (2)$$

Linearized Continuity Equation:

$$\frac{\partial n_{i1}}{\partial t} + n_{i0} \nabla \cdot \mathbf{v}_{i1} = 0$$

$$(-i\omega n_{i1}) + n_{i0} i k v_{iy} = 0$$

$$\frac{n_{i1}}{n_{i0}} = \frac{k}{\omega} v_{iy} \quad (3)$$

Substituting it in eq. (2)

$$V_{iy} = -\frac{i\omega a_i}{\omega} V_{ix} + \frac{\gamma_i k_B T_i}{m_i} \frac{k}{\omega} \left(\frac{k}{\omega} V_{iy} \right)$$

$$V_{iy} = -\frac{i\omega a_i}{\omega} V_{ix} + \frac{\gamma_i k_B T_i}{m_i} \frac{k^2}{\omega^2} V_{iy}$$

Suppose $\frac{\gamma_i k_B T_i}{m_i} \frac{k^2}{\omega^2} = A_i$

$$V_{iy} = -\frac{i\omega a_i}{\omega} V_{ix} + A_i V_{iy} \quad (4)$$

$$V_{iy} (1 - A_i) = -\frac{i\omega a_i}{\omega} V_{ix}$$

$$V_{iy} = \frac{-i\omega a_i}{\omega} (1 - A_i)^{-1} V_{ix} \quad (5)$$

Substituting eq. (5) in eq. (1)

$$V_{ix} = \frac{ie}{m_i \omega} E_x + \frac{ie B_0}{m_i \omega} \left(\frac{-i\omega a_i}{\omega} (1 - A_i)^{-1} V_{ix} \right)$$

$$V_{ix} = \frac{ie}{m_i \omega} E_x + \frac{i\omega a_i}{\omega} \left(\frac{-i\omega a_i}{\omega} (1 - A_i)^{-1} V_{ix} \right)$$

$$V_{ix} = \frac{ie}{m_i \omega} E_x + \frac{\omega a_i^2}{\omega^2} (1 - A_i)^{-1} V_{ix}$$

$$\left(1 - \frac{\omega a_i^2}{\omega^2} (1 - A_i)^{-1} \right) V_{ix} = \frac{ie E_x}{m_i \omega}$$

$$V_{ix} = \frac{ie}{m_i \omega} \left(1 - \frac{\omega a_i^2}{\omega^2} (1 - A_i)^{-1} \right)^{-1} E_x$$

$$V_{ix} = \frac{ie}{m_i \omega} \left(1 - \frac{\omega_{ci}^2}{\omega^2 (1 - A_i)} \right)^{-1} E_x \quad \text{--- (6)}$$

Similarly for electrons,

$$V_{ex} = -\frac{ie}{m_e \omega} \left(1 - \frac{\omega_{ce}^2}{\omega^2 (1 - A_e)} \right)^{-1} E_x \quad \text{--- (7)}$$

By using Maxwell's Equations, we have

$$(\omega^2 - c^2 k^2) E_x = -i 4 \pi \omega n_0 e (V_{ix} - V_{ex})$$

Substituting the values of V_{ix} & V_{ex} from eq. (6) & eq. (7) in above equation.

$$(\omega^2 - c^2 k^2) E_x = -i 4 \pi \omega n_0 e \left[\frac{ie}{m_i \omega} \frac{\omega^2 (1 - A_i)}{\omega^2 (1 - A_i) - \omega_{ci}^2} + \frac{ie}{m_e \omega} \frac{\omega^2 (1 - A_e)}{\omega^2 (1 - A_e) - \omega_{ce}^2} \right] E_x$$

$$\omega^2 - c^2 k^2 = \omega_{pi}^2 \frac{\omega^2 (1 - A_i)}{\omega^2 (1 - A_i) - \omega_{ci}^2} + \omega_{pe}^2 \frac{\omega^2 (1 - A_e)}{\omega^2 (1 - A_e) - \omega_{ce}^2} \quad \text{--- (8)}$$

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$$\text{As } A_i = \frac{\gamma_i k_B T_i}{m_i} \frac{k^2}{\omega^2}$$

$$\text{& } A_e = \frac{\gamma_e k_B T_e}{m_e} \frac{k^2}{\omega^2}$$

Since these are low frequency waves, the approximations of low frequency waves are

$$\text{i) } \omega^2 \ll \omega_{ci}^2$$

$$\omega^2 \ll \omega_{ce}^2$$

$$\text{ii) } \omega^2 \ll k^2 v_{the}^2$$

$$\text{iii) } v_{the}^2 > v_{thi}^2$$

Then eq. (8) becomes,

$$\omega^2 - c^2 k^2 = \omega_{pi}^2 \frac{\omega^2 (1 - A_i)}{-\omega_{ci}^2} + \omega_{pe}^2 \frac{\omega^2 (1 - A_e)}{-\omega_{ce}^2}$$

$$\omega^2 - c^2 k^2 = \omega p_i^2 \frac{\omega^2 (1 - A_i)}{-\omega^2 \epsilon_i} + \omega p_e^2 \frac{(-\omega^2 A_e)}{-\omega^2 \epsilon_e} \quad \therefore \omega^2 (1 - A_e) = \omega^2 \frac{\gamma_e k T_e}{m_e} \cdot \frac{k^2}{\omega^2}$$

$$\omega^2 - c^2 k^2 = \frac{4\pi n_0 e^2}{m_i} \frac{\omega^2 (1 - A_i)}{\frac{e^2 B_0^2}{m_i^2}} + \frac{4\pi n_0 e^2}{m_e} \frac{\omega^2 A_e}{\frac{e^2 B_0^2}{m_e^2}} \quad \therefore \omega^2 (1 - A_e) = \omega^2 \frac{\gamma_e k T_e}{m_e} \frac{k^2}{\omega^2}$$

$$\omega^2 - c^2 k^2 = \frac{4\pi n_0 m_i}{B_0^2} \omega^2 (1 - A_i) + \frac{4\pi n_0 m_e}{B_0^2} \omega^2 A_e$$

$$\omega^2 - c^2 k^2 = \frac{4\pi n_0 m_i}{B_0^2} \omega^2 \left[1 - \frac{\gamma_i k T_i}{m_i} \frac{k^2}{\omega^2} \right] + \frac{4\pi n_0 m_e}{B_0^2} \omega^2 \left[\frac{\gamma_e k T_e}{m_e} \frac{k^2}{\omega^2} \right]$$

$$\omega^2 - c^2 k^2 = -\frac{4\pi n_0 m_i}{B_0^2} \omega^2 + \frac{4\pi n_0 m_i \gamma_i k T_i}{B_0^2 m_i} \frac{k^2}{\omega^2} + \frac{4\pi n_0 m_e \gamma_e k T_e}{B_0^2 m_e} \frac{k^2}{\omega^2}$$

$$\omega^2 - c^2 k^2 = -\frac{4\pi n_0 m_i \omega^2}{B_0^2} + \frac{4\pi n_0 m_i \gamma_i k T_i}{B_0^2 m_i} k^2 + \frac{4\pi n_0 m_e \gamma_e k T_e}{B_0^2 m_e} k^2$$

$$\omega^2 - c^2 k^2 = -\frac{4\pi \beta_i}{B_0^2} \omega^2 + \frac{4\pi \beta}{B_0^2} k^2 \left(\frac{\gamma_i k T_i}{m_i} + \frac{\gamma_e k T_e}{m_e} \right) \quad \therefore n_0 m_i = \beta_i$$

Multiplying & dividing by "c²" on R.H.S,

$$\omega^2 - c^2 k^2 = -\frac{4\pi \beta_i c^2}{B_0^2 c^2} \omega^2 + \frac{4\pi \beta c^2}{B_0^2 c^2} k^2 \left(\frac{\gamma_i k T_i}{m_i} + \frac{\gamma_e k T_e}{m_e} \right)$$

$$\omega^2 - c^2 k^2 = -\frac{c^2 \omega^2}{V_A^2} + \frac{c^2 k^2}{V_A^2} \cdot V_s^2$$

$$\frac{\omega^2 + c^2 \omega^2}{V_A^2} = \frac{c^2 k^2 + c^2 k^2 \cdot V_s^2}{V_A^2}$$

$$\omega^2 \left(1 + \frac{c^2}{V_A^2} \right) = k^2 \left(c^2 - \frac{c^2}{V_A^2} V_s^2 \right)$$

$$\omega^2 \left(1 + \frac{c^2}{V_A^2} \right) = c^2 k^2 \left(1 - \frac{V_s^2}{V_A^2} \right)$$

$$\frac{\omega^2}{k^2} = \frac{c^2 \left(1 - \frac{V_s^2}{V_A^2} \right)}{1 + \frac{c^2}{V_A^2}}$$

$$\frac{\omega^2}{k^2} = \frac{c^2 (V_A^2 + V_S^2)}{V_A^2}$$

$$\frac{\omega^2}{k^2} = \frac{c^2 (V_A^2 + V_S^2)}{(V_A^2 + c^2)}$$

This is the dispersion relation for magnetosonic wave.

If there is no magnetic field then V_A become zero so, we again reach to the dispersion relation of ion sound waves.

Phase Velocity:-

It is more than the Alfvén velocity so magnetosonic waves are also known as the fast hydrodynamic waves.